

A LOW-COMPLEXITY RELAY TRANSMIT STRATEGY FOR TWO-WAY RELAYING WITH MIMO AMPLIFY AND FORWARD RELAYS

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Abstract — In this paper we consider two-way relaying with a MIMO amplify and forward (AF) relay. In the literature, the relay amplification matrix which maximizes the sum rate in two-way relaying is not known for the general MIMO case. However, the maximization of the channels' Frobenius norms is easily achieved via the Algebraic Norm-Maximizing (ANOMAX) transmit strategy. While this scheme provides a significant improvement in the received signals' strengths, it does not reach the full multiplexing gain for high SNRs due to its low rank nature. Therefore, we propose a simple strategy to restore the rank while preserving the same subspaces via an optimization over the profile of the singular values. The resulting scheme is called rank-restored ANOMAX (RR-ANOMAX).

The main benefit of this approach is that the computational complexity is very low. Moreover, its performance is very close to the one-way upper-bound which is obtained by considering the two transmission directions as independent one-way relaying channels.

Index Terms— Two-Way Relaying, Amplify and Forward, Beamforming

1. INTRODUCTION

Relaying is considered as a candidate technology for future mobile communication systems. In particular, two-way relaying [3] is known as a technique which uses the radio resources in a particularly efficient manner. In this scheme, two subsequent time slots are used to establish a bidirectional transmission between two terminals: In the first slot, both terminals transmit to the relay, in the second slot the relay transmits back to both terminals. This compensates the spectral efficiency loss in one-way relaying due to the half duplex constraint of the relay [4, 2]. We consider amplify and forward (AF) relays which retransmit an amplified version of their received signal since these cause less transmission delay and require lower hardware complexity than decode and forward (DF) relays.

It is desirable to find the relay transmit strategy which maximizes the (weighted) sum rate of both users. In [10], the capacity region and an iterative scheme to compute the optimal relay amplification matrix are discussed for the special case that the terminals have a single receive antenna. For the same case, we have introduced a significantly simpler scheme to achieve the maximum sum rate called GERMS (Generalized Eigenvector Based Rate-Maximizing Transmit Strategy for Single-Antenna Terminals) in [6].

However, in this paper we consider the general MIMO case, where each terminal may have more than one antenna. The existing rate-optimal approaches for single-antenna terminals from [10] and [6] cannot be extended to multiple antennas.

For this case, [9] proposes the dual channel matching (DCM) scheme as a suboptimal approach to reduce the complexity of the optimization problem. Moreover, in [8], ZF and MMSE transceivers

are introduced for the case where the number of antennas at the relay is greater or equal to the sum of the number of antennas at the user terminals. However, these schemes suffer from a loss in energy of the desired signal since the interference is forced to zero at the relay.

The algebraic norm-maximizing (ANOMAX) transmit strategy has been introduced in [5]. ANOMAX maximizes the weighted sum of the Frobenius norms of the effective channels without requiring any iterative procedures. While ANOMAX results in a significant improvement in the received signals' strengths, it does not achieve the full multiplexing gain for high SNRs due to the low-rank nature of the solution.

Therefore, in this paper, we propose to restore the rank of the relay amplification matrix by optimizing the sum rate over the profile of the singular values while keeping the left and the right singular vectors fixed. The resulting strategy called RR-ANOMAX (rank-restored ANOMAX) combines the good performance of ANOMAX for low SNRs with the rank requirement for high SNRs. We first propose an exhaustive search over the singular values which requires an $M_R - 1$ dimensional numerical optimization, where M_R denotes the number of antennas at the relay station. As an alternative with a lower computational complexity, we introduce a closed-form approximation to the optimal profile of singular values called WF-RR-ANOMAX, since it is inspired by the water filling (WF) algorithm¹.

We show via simulations that WF-RR-ANOMAX achieves nearly the same sum rate as the exhaustive search needed for RR-ANOMAX. Since the achievable rate region in two-way relaying is not known for the general MIMO case, we compare the different approaches to a "one-way upper bound" which is obtained by considering the two transmission directions as separate one-way links. As we show in Section 5, RR-ANOMAX and WF-RR-ANOMAX yield a sum rate very close to this upper bound even though this bound is not achievable in general.

2. SYSTEM DESCRIPTION

The two-way relaying system we investigate in this paper is shown in Figure 1. An intermediate relay station RS assists the bidirectional transmission between two user terminals UT₁ and UT₂. The relay is equipped with M_R antennas, the terminals UT₁ and UT₂ have M_1 and M_2 antennas, respectively.

The data transmission is performed in two subsequent time slots. In the first time slot, both terminals transmit to the relay, where their transmissions interfere. The signal received at the relay can be written as

$$\mathbf{r} = \mathbf{H}_1 \cdot \mathbf{x}_1 + \mathbf{H}_2 \cdot \mathbf{x}_2 + \mathbf{n}_R,$$

¹Even though WF is computed iteratively, the number of iterations is not data-dependent but only a function of the number of antennas and hence strictly bounded. Thus, in contrast to schemes which iterate until a threshold-based stopping criterion is fulfilled, it can be called closed-form.

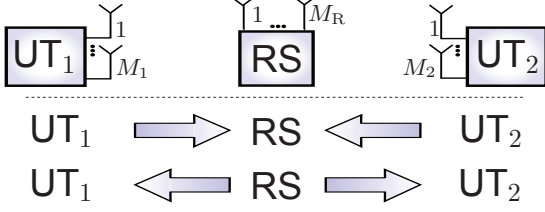


Fig. 1. Two-way relaying system model.

where $\mathbf{H}_1 \in \mathbb{C}^{M_R \times M_1}$ and $\mathbf{H}_2 \in \mathbb{C}^{M_R \times M_2}$ represent the flat fading MIMO channels between the terminals and the relay, $\mathbf{x}_1 \in \mathbb{C}^{M_1 \times 1}$ and $\mathbf{x}_2 \in \mathbb{C}^{M_2 \times 1}$ are the transmitted signals from the terminals, and the vector \mathbf{n}_R contains the noise component at the relay.

In the second time slot, the relay amplifies \mathbf{r} and transmits the resulting signal $\bar{\mathbf{r}}$ via

$$\bar{\mathbf{r}} = \gamma \cdot \mathbf{G} \cdot \mathbf{r}, \quad (1)$$

where $\mathbf{G} \in \mathbb{C}^{M_R \times M_R}$ is the relay amplification matrix. Note that \mathbf{G} is normalized such that $\|\mathbf{G}\|_F = 1$, where $\gamma \in \mathbb{R}^+$ represents a scalar amplification factor that guarantees that the relay transmit power constraint of $P_{T,R}$ is satisfied.

The terminals receive the amplified signal $\bar{\mathbf{r}}$ from the relay via their reverse channels. We assume that reciprocity holds and therefore write the received signals $\mathbf{y}_1 \in \mathbb{C}^{M_1 \times 1}$ and $\mathbf{y}_2 \in \mathbb{C}^{M_2 \times 1}$ as

$$\mathbf{y}_1 = \gamma \cdot \mathbf{H}_{1,1}^{(e)} \cdot \mathbf{x}_1 + \gamma \cdot \mathbf{H}_{1,2}^{(e)} \cdot \mathbf{x}_2 + \tilde{\mathbf{n}}_1 \quad (2)$$

$$\mathbf{y}_2 = \gamma \cdot \mathbf{H}_{2,2}^{(e)} \cdot \mathbf{x}_2 + \gamma \cdot \mathbf{H}_{2,1}^{(e)} \cdot \mathbf{x}_1 + \tilde{\mathbf{n}}_2, \quad (3)$$

where $\tilde{\mathbf{n}}_i = \gamma \cdot \mathbf{H}_i^T \cdot \mathbf{G} \cdot \mathbf{n}_R + \mathbf{n}_i$ represents the effective noise at terminal i which consists of the i -th terminal's own noise and the forwarded relay noise. Moreover, we have defined the effective channels

$$\mathbf{H}_{i,j}^{(e)} = \mathbf{H}_i^T \cdot \mathbf{G} \cdot \mathbf{H}_j, \quad i, j = 1, 2. \quad (4)$$

As it is evident from (2) and (3) the terminals receive the desired signals from the other terminals via the effective channels $\mathbf{H}_{1,2}^{(e)}$ and $\mathbf{H}_{2,1}^{(e)}$ and their self-interference via $\mathbf{H}_{1,1}^{(e)}$ and $\mathbf{H}_{2,2}^{(e)}$, respectively. Since each terminal knows its own transmitted signal, the self-interference can be subtracted if channel knowledge is available. This step is often referred to as Analogue Network Coding (ANC) [1]. In this paper, we assume that the channel knowledge is perfect and only consider the desired signal as well as the effective noise terms.

3. RANK-RESTORED ANOMAX

In [5] the relay amplification matrix was chosen to maximize the (weighted) sum of the Frobenius norms of the effective channels via the ANOMAX scheme. ANOMAX provides a solution for

$$\mathbf{G}_{\text{ANOMAX}} = \arg \max_{\mathbf{G}, \|\mathbf{G}\|_F=1} \left\{ \left\| \mathbf{H}_{1,2}^{(e)} \right\|_F^2 + \left\| \mathbf{H}_{2,1}^{(e)} \right\|_F^2 \right\} \quad (5)$$

and is obtained from $\text{vec}\{\mathbf{G}_{\text{ANOMAX}}\} = \mathbf{u}_1^*$, where the $*$ represents complex conjugation. Here, \mathbf{u}_1 is the dominant left singular vector of the matrix \mathbf{K} given by

$$\mathbf{K} = [\mathbf{H}_2 \otimes \mathbf{H}_1, \mathbf{H}_1 \otimes \mathbf{H}_2] \in \mathbb{C}^{M_R^2 \times 2 M_1 M_2}, \quad (6)$$

where \otimes represents the Kronecker product. Note that for simplicity we have ignored the weighting coefficient β which is equivalent to setting $\beta = 0.5$. It was observed in [5] that ANOMAX significantly improves the received signals' strengths for a single-stream transmission. However, it tends to concentrate most of the energy on the

dominant singular value. In particular, the resulting \mathbf{G} is a rank-2 matrix where the second singular value is significantly smaller than the dominant one. This is detrimental for the sum rate since for high SNRs, the full spatial multiplexing gain can only be achieved if all spatial modes become active. This requires that the ranks of $\mathbf{H}_{1,2}^{(e)}$ and $\mathbf{H}_{2,1}^{(e)}$ are at least $\min\{M_1, M_2, M_R\}$.

To achieve the full spatial multiplexing gain, the rank of \mathbf{G} has to be increased. This can be achieved by properly adjusting the singular values of \mathbf{G} . We propose to leave the singular vectors of \mathbf{G} intact and to adjust only the M_R singular values. As we show in the sequel, this requires an $M_R - 1$ dimensional optimization, since \mathbf{G} is normalized. To this end, let the SVD of $\mathbf{G}_{\text{ANOMAX}}$ be given by

$$\mathbf{G}_{\text{ANOMAX}} = \mathbf{U}_A \cdot \mathbf{\Sigma}_A \cdot \mathbf{V}_A^H. \quad (7)$$

Then the singular value profile can be adjusted via the vector $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \dots, \sigma_{M_R}]^T$ by defining

$$\mathbf{G}(\boldsymbol{\sigma}) = \mathbf{U}_A \cdot \text{diag}\{\boldsymbol{\sigma}\} \cdot \mathbf{V}_A^H, \quad (8)$$

where $\text{diag}\{\boldsymbol{\sigma}\}$ is a diagonal matrix containing the elements of the vector $\boldsymbol{\sigma}$ on its main diagonal. Our new optimization problem therefore takes the following form

$$\max_{\boldsymbol{\sigma}} r(\mathbf{G}(\boldsymbol{\sigma})), \quad \text{s.t. } \|\boldsymbol{\sigma}\|_2 = 1 \text{ and } \sigma_1 \geq \sigma_2 \geq \dots \geq 0 \quad (9)$$

where $r(\mathbf{G}(\boldsymbol{\sigma}))$ denotes the sum rate achievable with the relay amplification matrix $\mathbf{G}(\boldsymbol{\sigma})$, i.e., the sum of the capacities of $\mathbf{H}_1^T \cdot \mathbf{G}(\boldsymbol{\sigma}) \cdot \mathbf{H}_2$ and $\mathbf{H}_2^T \cdot \mathbf{G}(\boldsymbol{\sigma}) \cdot \mathbf{H}_1$. The constraints stem from the fact that $\mathbf{G}(\boldsymbol{\sigma})$ is normalized and the singular values are ordered and non-negative.

Note that in the special case $\sigma_i = 1/\sqrt{M_R}, \forall i$, the matrix $\sqrt{M_R} \cdot \mathbf{G}(\boldsymbol{\sigma})$ becomes unitary. As shown in [11] this represents the best approximation of $\sqrt{M_R} \cdot \mathbf{G}_{\text{ANOMAX}}$ by a unitary matrix in the Frobenius norm sense, commonly referred to as the Procrustes approximation.

The optimization problem in (9) can be simplified by taking into account the nature of our parameters. First of all, the norm constraint on $\boldsymbol{\sigma}$ can be used to reduce the search space to $M_R - 1$ dimensions by optimizing over $\bar{\boldsymbol{\sigma}} = [\sigma_2/\sigma_1, \dots, \sigma_{M_R}/\sigma_1]^T \in \mathbb{R}^{M_R-1}$, where each element of $\bar{\boldsymbol{\sigma}}$ is in $[0, 1]$. Secondly, the search space for $\bar{\boldsymbol{\sigma}}$ can be further reduced by taking into account that the singular values are ordered, i.e., the i -th element of $\bar{\boldsymbol{\sigma}}$ is optimized in the interval between 0 and the current value of the $(i-1)$ -th element of $\bar{\boldsymbol{\sigma}}$.

4. WF-BASED HEURISTIC

RR-ANOMAX requires the optimization over $M_R - 1$ real-valued parameters, which can become cumbersome for larger values of M_R . A typical result of RR-ANOMAX is depicted in Figure 2, where we consider uncorrelated Rayleigh fading channels with $M_1 = M_2 = 6$, $M_R = 3$ and depict the resulting profile of the squared singular values obtained via RR-ANOMAX. We observe that for low SNRs, a low-rank solution is obtained and for high SNRs, all singular values become equal. We have found a similar trend in all other scenarios that have been investigated.

It is therefore possible to replace the optimization procedure by a closed-form water-filling (WF) based heuristic. The resulting singular values do not perfectly match the ones obtained via RR-ANOMAX. However, since the cost function is not very sensitive to small changes in the singular values, the sum rate achieved via this heuristic is always very close to RR-ANOMAX, which we demonstrate numerically in Section 5.

Our WF-based closed-form solution chooses σ_k according to

$$\sigma_k^2 = \left(\mu - \frac{P_{N,R}}{\lambda_k} \right)_+, \quad k = 1, 2, \dots, r, \quad (10)$$

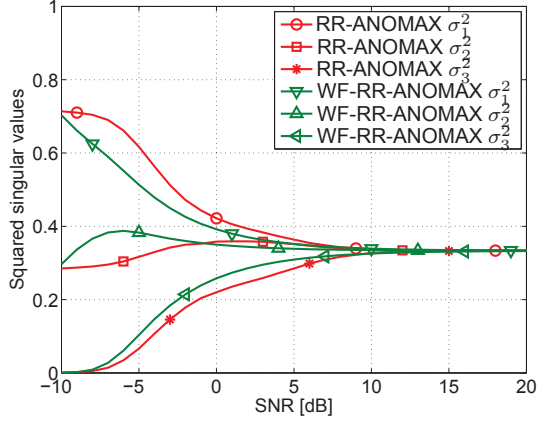


Fig. 2. Profile of squared singular values optimized via RR-ANOMAX and WF-RR-ANOMAX for $M_1 = M_2 = 6$, $M_R = 3$.

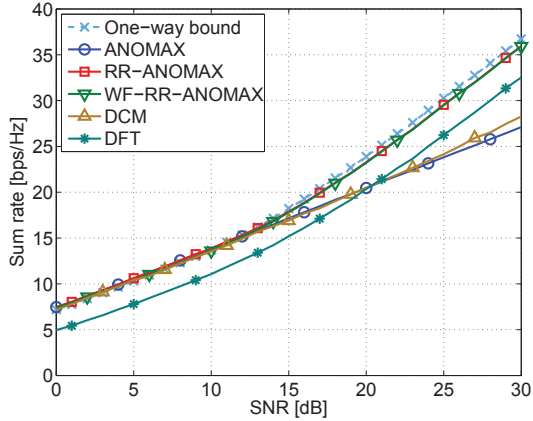


Fig. 3. Sum rate vs. SNR for $M_1 = M_2 = M_R = 3$, $\rho_R = 0.9$, and $\rho_1 = \rho_2 = 0$.

where $P_{N,R}$ represents the noise power at the relay, $(x)_+ = \max\{0, x\}$, and μ is the water level such that $\|\sigma\|_2 = 1$. Here, λ_k represents the virtual eigenvalue profile which we can compute via

$$\lambda_k = (\sigma_{1,k} + \delta) \cdot (\sigma_{2,k} + \delta), \quad k = 1, 2, \dots, r, \quad (11)$$

where δ is a positive constant to assure that we obtain r non-zero eigenvalues for high SNRs ($\delta = 1$ is used in the simulations). The rank r is chosen as $r = \min\{M_R, \min\{M_1, M_2\} + 1\}$. Comparing the profile of the singular values obtained via WF-RR-ANOMAX and RR-ANOMAX in Figure 2 we observe that they follow a similar trend. Note that WF-RR-ANOMAX is just one example for a possible heuristic.

5. SIMULATION RESULTS

In this section we present the results of numerical computer simulations to demonstrate the achievable rate with the RR-ANOMAX approach developed in the previous sections.

We assume that both terminals and the relay have a transmit power of 1 and experience zero mean circularly symmetric complex Gaussian noise with variance P_N . Consequently, the SNR is defined as P_N^{-1} . Moreover, we consider Rayleigh fading channels with different antenna configurations at the transmitter and the receiver. In the case of correlated Rayleigh fading channels, we assume the Kro-

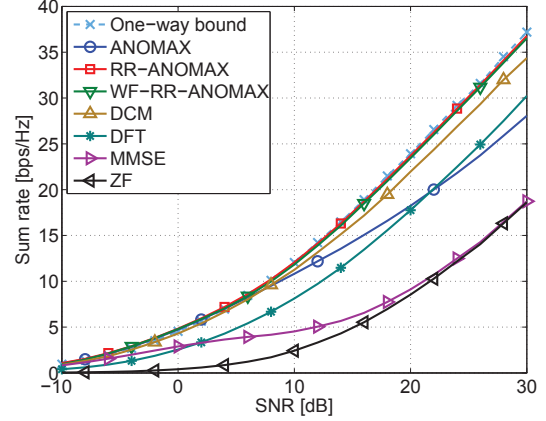


Fig. 4. Sum rate vs. SNR for $M_1 = M_2 = 2$, $M_R = 4$, and $\rho_1 = \rho_2 = \rho_R = 0$.

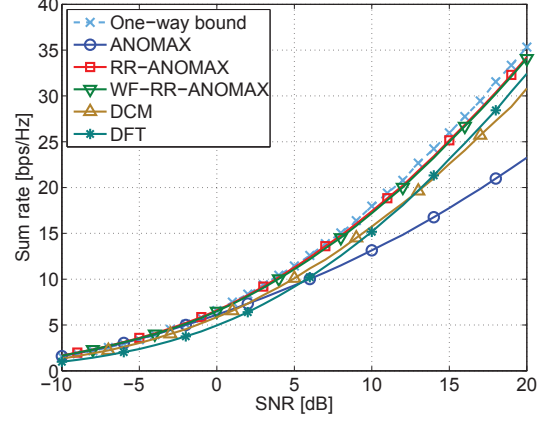


Fig. 5. Sum rate vs. SNR for $M_1 = M_2 = M_R = 4$, and $\rho_1 = \rho_2 = \rho_R = 0$.

necker correlation model which can be expressed as

$$\mathbb{E}\{\mathbf{H}_i \cdot \mathbf{H}_i^H\} = \mathbf{R}_R, \quad i = 1, 2, \quad (12)$$

$$\mathbb{E}\{\mathbf{H}_i^H \cdot \mathbf{H}_i\} = \mathbf{R}_i, \quad i = 1, 2, \quad (13)$$

where $\mathbf{R}_R \in \mathbb{C}^{M_R \times M_R}$ and $\mathbf{R}_i \in \mathbb{C}^{M_i \times M_i}$ represent the spatial correlation matrices at the relay and at the user terminal i , respectively. For simplicity, the matrices \mathbf{R}_R and \mathbf{R}_i are chosen such that their main diagonal elements are equal to one and the magnitude of all off-diagonal elements is equal to ρ_R and ρ_i , respectively.

We display the normalized sum rate in Bits/s/Hz which is the sum of the capacities of the two effective channels $\mathbf{H}_1^T \cdot \mathbf{G} \cdot \mathbf{H}_2$ and $\mathbf{H}_2^T \cdot \mathbf{G} \cdot \mathbf{H}_1$ for various choices of \mathbf{G} . Here, RR-ANOMAX refers to the solution of (9) via an exhaustive search over $\bar{\sigma}$ and WF-RR-ANOMAX denotes the water filling based closed-form solution. For the curve labeled ‘‘DFT’’ we do not need channel knowledge at the relay and set \mathbf{G} to a DFT matrix of size $M_R \times M_R$ normalized to unit Frobenius norm. Moreover, the original ANOMAX solution from [5] and the dual channel matching (DCM) strategy from [9] are shown for comparison. For the case where $M_R \geq M_1 + M_2$ we also depict the ZF and the MMSE schemes introduced in [8].

Since the achievable rate region in two-way relaying is not known for the general MIMO case, we compare the different approaches to a ‘‘one-way upper bound’’. This bound is obtained by

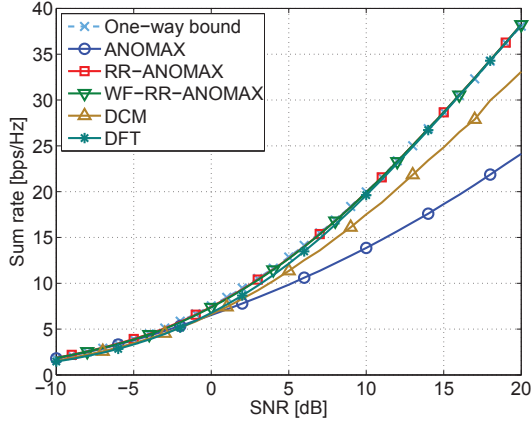


Fig. 6. Sum rate vs. SNR for $M_1 = M_2 = 6$, $M_R = 3$, and $\rho_1 = \rho_2 = \rho_R = 0$.

first considering only one of the transmission directions, computing the rate-optimal relay amplification matrix for this link according to [7], and then determining the capacity of this one-way link. Then, the same procedure is repeated for the other transmission direction and the resulting capacities are added. The corresponding curve is labeled “One-way bound” in our simulation results. Note that this bound is in general not achievable but we expect the result to be close due to the nature of ANC [1].

Figure 3 displays the case where $M_1 = M_2 = M_R = 3$. Moreover, in this scenario we introduce spatial correlation at the relay by setting $\rho_R = 0.9$ but no spatial correlation at the terminals ($\rho_1 = \rho_2 = 0$). We observe that ANOMAX performs well for low SNRs where only a single stream is used since in this case the improvement in signal strength obtained via ANOMAX is vital. However, for higher SNRs, due to the low-rank nature of ANOMAX, it fails to achieve the required multiplexing gain. Therefore, the simple DFT solution outperforms ANOMAX at a certain SNR, even though it does not take channel state information into account. Via RR-ANOMAX, we can restore the required rank for high SNRs while preserving the good performance of ANOMAX for low SNRs. Comparing the exhaustive search solution RR-ANOMAX with the sub-optimal closed-form water filling approach WF-RR-ANOMAX, we observe that the latter performs almost identically well even though its complexity is much smaller. Moreover, both are very close to the one-way upper bound and better than DCM.

A similar conclusion is drawn from Figure 4, where we set $M_1 = M_2 = 2$, $M_R = 4$, and consider uncorrelated Rayleigh fading. Here, the improvement in received signal strength obtained via ANOMAX is even more pronounced since there are more degrees of freedom in the relay amplification matrix to control less parameters in the effective channel matrices. Still, for high SNRs, ANOMAX suffers from its low-rank nature and is hence outperformed by the DFT. The RR-ANOMAX scheme requires a 3-dimensional optimization which can already become quite cumbersome. However, comparing it to WF-RR-ANOMAX, we observe again that WF-RR-ANOMAX performs almost identically well and both are close to the one-way upper bound. Since in this scenario, the condition $M_R \geq M_1 + M_2$ from [8] is fulfilled, we can depict the ZF and MMSE approaches as well. The comparison is a bit unfair since ZF and MMSE suppress the self-interference at the relay so that the terminals do not have to subtract it themselves (i.e., they do not require channel knowledge). As it is evident from the sum rate performance, this form of interference mitigation causes a signif-

icant rate loss which is due to the noise enhancement at the relay. Therefore, the self-interference should always be canceled by the terminals themselves.

Finally, Figures 5 and 6 depict the result of choosing $M_1 = M_2 = M_R = 4$ and $M_1 = M_2 = 6$, $M_R = 3$, respectively. Again, uncorrelated Rayleigh fading is assumed ($\rho_R = \rho_1 = \rho_2 = 0$). The observations are similar to the previous results: RR-ANOMAX successfully restores the rank of ANOMAX which is needed to provide the full spatial multiplexing gain for high SNRs. Moreover, the sub-optimal WF-RR-ANOMAX solution performs almost as good as the exhaustive search needed for RR-ANOMAX, both are very close to the one-way upper bound, and better than DCM.

6. CONCLUSIONS

In this paper we propose a low-complexity solution for the relay amplification matrix in a MIMO two-way relaying system with amplify and forward relays. We start from the ANOMAX solution that maximizes the effective channels’ Frobenius norms and hence significantly improves the signal strength which is especially important if the SNR is very low. However, due to the low-rank nature of ANOMAX, it cannot provide the full spatial multiplexing gain for high SNRs and is therefore outperformed by simple full-rank solutions. Consequently, we propose to restore the rank via RR-ANOMAX by choosing the relay amplification matrix such that the singular vectors are the same as the ones used for ANOMAX, but the singular values are changed to provide the maximum sum rate. This requires an exhaustive search over $M_R - 1$ parameters.

Finally, we reduce the complexity even further by proposing a closed-form solution to find the singular values which is inspired by the water filling algorithm called WF-RR-ANOMAX. While this approach provides suboptimal singular values, its sum rate performance is very close to the exhaustive search solution in numerical simulations. Moreover, both approaches are very close to the “one-way bound” which is an upper bound on the sum rate obtained by treating the two transmission directions as independent one-way relaying channels.

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