Iterative Sequential GSVD (I-S-GSVD) based Prewhitening for Multidimensional HOSVD based Subspace Estimation without Knowledge of the Noise Covariance Information

João Paulo C. L. da Costa[1]*, Florian Roemer, and Martin Haardt*
Ilmenau University of Technology, Communications Research Laboratory
P.O. Box 100565, D-98684 Ilmenau, Germany.
{joaopaulo.dacosta,florian.roemer,martin.haardt}@tu-ilmenau.de
http://www.tu-ilmenau.de/crl

Abstract **— Recently, the Sequential GSVD (S-GSVD) based prewhitening scheme has been proposed to improve R-dimensional subspace-based parameter estimation schemes in the presence of colored noise or interference with Kronecker structure. To apply the S-GSVD, second order statistics of the noise should be estimated, e.g., via samples captured in the absence of the desired signal components.**

In this contribution, we propose the Iterative Sequential Generalized Singular Value Decomposition (I-S-GSVD) based prewhitening scheme for multidimensional HOSVD based subspace estimation when information about the noise statistics is not available. Even without the availability of samples in the absence of the desired signals components, it is possible to obtain the prewhitening correlation factors and the signal parameters in an iterative way using a deterministic algorithm in combination with the S-GSVD. This combination constitutes our proposed I-S-GSVD. Finally, the I-S-GSVD inherits the computational efficiency from the S-GSVD compared to matrix based prewhitening schemes.

I. INTRODUCTION

High-resolution parameter estimation from R-dimensional signals is a task required for a variety of applications, such as estimating the multidimensional parameters of the dominant multipath components from MIMO channel sounder measurements, radar, sonar, seismology, and medical imaging.

As shown in [1], subspace-based parameter estimation schemes can be significantly improved via the S-GSVD based prewhitening technique in environments with multidimensional colored noise with Kronecker structure, which is found in certain EEG applications [2] as well as in certain MIMO applications [3].

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For many EEG and MIMO applications, noise samples can only be collected in the presence of the desired signal components. Therefore, in order to improve the subspace-based parameter estimation in such problems, we propose a multidimensional prewhitening scheme called Iterative S-GSVD (I-S-GSVD). The I-S-GSVD has the low complexity of the S-GSVD and at the same time, it provides a performance close to the one obtained in the case that the second order statistics of the noise are estimated from samples where the desired signal components are absent.

In [1], the parameter estimation accuracy using matrix based prewhitening schemes [4], [5], [6] is significantly degraded in the case where only a small number of samples is available in the absence of the desired signal components. As shown in [1], the S-GSVD based prewhitening scheme can also handle such a case successfully.

II. DATA MODEL

We consider the superposition of d planar wavefronts captured by an R-D array at N subsequent time instants. In the r-th dimension of the R-D array, there are M_r sensors. Thus, the measurements obey the following model [7]

$$\begin{aligned} \boldsymbol{\mathcal{X}} &= \boldsymbol{\mathcal{A}} \times_{R+1} \boldsymbol{S}^{\mathrm{T}} + \boldsymbol{\mathcal{N}}^{(\mathrm{c})}, \quad \text{where} \\ \boldsymbol{\mathcal{A}} &= \boldsymbol{\mathcal{I}}_{R+1,d} \times_1 \boldsymbol{A}^{(1)} \ldots \times_R \boldsymbol{A}^{(R)}. \end{aligned} \qquad (1)$$

The matrix $\boldsymbol{A}^{(r)} \in \mathbb{C}^{M_r \times d}$ denotes the array steering matrix in the r-th mode with $r = 1, \ldots, R$, the factor matrix $\boldsymbol{S} \in \mathbb{C}^{d \times N}$ contains the symbols $s_i(n)$, and the tensor $\boldsymbol{\mathcal{N}}^{(\mathrm{c})} \in \mathbb{C}^{M_1 \times M_2 \times \ldots \times M_R \times N}$ contains the ZMCSCG (zero-mean circularly-symmetric complex Gaussian) noise samples with variance σ_n^2. We define $\boldsymbol{\mathcal{I}}_{R+1,d} \in \mathbb{R}^{d \times d \times \ldots \times d}$ as the identity tensor with $R+1$ dimensions. The elements of $\boldsymbol{\mathcal{I}}_{R+1,d}$ are equal to 1 when all indices are equal and 0 otherwise. In (1), the operator $\times_r$ stands for the r-mode product, which is defined according to [8]. The r-mode unfolding of $\boldsymbol{\mathcal{A}}$ is represented by $[\boldsymbol{\mathcal{A}}]_{(r)}$ and it is the matrix form of $\boldsymbol{\mathcal{A}}$ varying the r-th index along the rows and stacking all the other indices along the columns of $[\boldsymbol{\mathcal{A}}]_{(r)}$ in the same order as

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in [8]. The superscript $^{\mathrm{T}}$ stands for transposition and other superscripts used here are $^{\mathrm{H}}, ^{-1}$, and $^{+}$, which mean Hermitian transposition, matrix inversion, and the Moore-Penrose pseudo inverse of matrices, respectively.

We represent the i-th column of $\boldsymbol{A}^{(r)}$ in (1) as $\boldsymbol{a}_i^{(r)}$, which has a Vandermonde structure as function of $e^{j\cdot\mu_i^{(r)}}$, where $\mu_i^{(r)}$ are the spatial frequencies of the i-th source $(i = 1, \ldots, d)$ in the r-th dimension $(r = 1, \ldots, R)$. Our objective is to improve the estimate of all the spatial frequencies $\mu_i^{(r)}$ from $\boldsymbol{\mathcal{X}}$ in the presence of colored noise or interference with a Kronecker structure. The model order is assumed known. For efficient multi-dimensional model order selection schemes, the reader is referred to [9], [10], [11]. In addition, in case that the colored noise presents some special structure, the deterministic prewhitening scheme presented in [12] can be applied in conjunction with the proposed I-S-GSVD.

As in [1], the multidimensional colored noise is assumed to have a Kronecker structure, which can be written as

$$\left[\boldsymbol{\mathcal{N}}^{(\mathrm{c})}\right]_{(R+1)} = [\boldsymbol{\mathcal{N}}]_{(R+1)} \cdot (\boldsymbol{L}_1 \otimes \boldsymbol{L}_2 \otimes \ldots \otimes \boldsymbol{L}_R)^{\mathrm{T}}, \quad (2)$$

where $\boldsymbol{\mathcal{N}} \in \mathbb{C}^{M_1 \times M_2 \ldots \times N}$ is a white noise tensor with ZMCSCG elements, $\otimes$ represents the Kronecker product and $\boldsymbol{L}_i \in \mathbb{C}^{M_i \times M_i}$ is the correlation factor of the i-th dimension of the colored noise tensor $\boldsymbol{\mathcal{N}}^{(\mathrm{c})}$. Similarly to [1], we can rewrite (2) by using the n-mode products in the following fashion

$$\boldsymbol{\mathcal{N}}^{(\mathrm{c})} = \boldsymbol{\mathcal{N}} \times_1 \boldsymbol{L}_1 \times_2 \boldsymbol{L}_2 \ldots \times_R \boldsymbol{L}_R. \quad (3)$$

The noise covariance matrix in the i-th mode $\boldsymbol{W}_i$ is defined as

$$\mathrm{E}\left\{\left[\boldsymbol{\mathcal{N}}^{(\mathrm{c})}\right]_{(i)} \cdot \left[\boldsymbol{\mathcal{N}}^{(\mathrm{c})}\right]_{(i)}^{\mathrm{H}}\right\} = \alpha \cdot \boldsymbol{W}_i = \alpha \cdot \boldsymbol{L}_i \cdot \boldsymbol{L}_i^{\mathrm{H}}, \quad (4)$$

where α is a normalization constant, such that $\mathrm{tr}(\boldsymbol{L}_i \cdot \boldsymbol{L}_i^{\mathrm{H}}) = M_i$. The equivalence between (2), (3), and (4) is shown in [1], and it is the basis for the S-GSVD [1].

III. Iterative Sequential GSVD (I-S-GSVD)

In this section, we present the proposed deterministic algorithm to compute the I-S-GSVD. Here we apply the prewhitening correlation factor estimation (PCFE) iteratively to compute $\hat{\boldsymbol{L}}_i$, an estimate of $\boldsymbol{L}_i$. The PCFE is based on estimating $\boldsymbol{W}_i$ by dropping the expectation operator in (4) and then factorizing this estimate to obtain $\hat{\boldsymbol{L}}_i$, e.g., via a Cholesky factorization. More details about the PCFE are provided in [1].

Inspired by [13], where a deterministic version of the Expectation Maximization (EM) algorithm is proposed, we propose the *iterative S-GSVD (I-S-GSVD)*. Similarly to the deterministic EM [13], a certain structure of the data is assumed in our scheme. Our proposed multidimensional prewhitening scheme is applied to improve the estimation of the spatial frequencies. Before starting the deterministic algorithm, first let us define K and k as the maximum number of iterations in the iterative approach and the iteration index, respectively.

0) Set $k = 1$.
1) Given $\boldsymbol{\mathcal{X}}$, compute the subspace tensor $\boldsymbol{\mathcal{U}}^{[s]} \in \mathbb{C}^{M_1 \times \ldots \times M_R \times d}$ via a truncated HOSVD [8]-based low-rank approximation according to [7].
2) Given $\boldsymbol{\mathcal{U}}^{[s]}$, estimate the spatial frequencies $\hat{\mu}_{k,i}^{(r)}$ for $r = 1, \ldots, R$ and for $i = 1, \ldots, d$, for instance, via R-D Standard-Tensor ESPRIT (R-D STE) [7]. Note that $\hat{\mu}_{k,i}^{(r)}$ is the estimate of $\mu_i^{(r)}$ in the k-th iteration.
3) From $\hat{\mu}_{k,i}^{(r)}$, compute $\hat{\boldsymbol{\mathcal{A}}}$ according to the structure of our data model in (1). Using $\boldsymbol{\mathcal{X}}$ and $\hat{\boldsymbol{\mathcal{A}}}$, calculate $\hat{\boldsymbol{S}} = \left([\boldsymbol{\mathcal{X}}]_{(R+1)} \cdot \left[\hat{\boldsymbol{\mathcal{A}}}\right]_{(R+1)}^{+}\right)^{\mathrm{T}}$.
4) Given $\hat{\boldsymbol{\mathcal{A}}}$ and $\hat{\boldsymbol{S}}$, the estimate of the noise tensor $\hat{\boldsymbol{\mathcal{N}}}^{(\mathrm{c})}$ can be compute via $\hat{\boldsymbol{\mathcal{N}}}^{(\mathrm{c})} = \boldsymbol{\mathcal{X}} - \hat{\boldsymbol{\mathcal{A}}} \times_{R+1} \hat{\boldsymbol{S}}^{\mathrm{T}}$.
5) From $\hat{\boldsymbol{\mathcal{N}}}^{(\mathrm{c})}$, we can estimate via PCFE the correlation factors $\hat{\boldsymbol{L}}_r$ according to [1].
6) Increment k.
7) According to [1], compute the subspace tensor $\boldsymbol{\mathcal{U}}^{[s]} \in \mathbb{C}^{M_1 \times \ldots \times M_R \times d}$ via the S-GSVD II of $\boldsymbol{\mathcal{X}}$ and $\hat{\boldsymbol{L}}_r$, for $r = 1, \ldots, R$.
8) Given $\boldsymbol{\mathcal{U}}^{[s]}$, estimate the spatial frequencies $\hat{\mu}_{k,i}^{(r)}$ for $r = 1, \ldots, R$ and for $i = 1, \ldots, d$ for instance via R-D Standard-Tensor ESPRIT (R-D STE) [7].
9) Compute the root mean square change (RMSC) of the spatial frequencies estimates $E_\mu^{(k)} = \sqrt{\sum_{r=1}^{R}\sum_{i=1}^{d}\left(\hat{\mu}_{k,i}^{(r)} - \hat{\mu}_{k-1,i}^{(r)}\right)^2}$. If the RMSC $E_\mu^{(k)}$ is zero or smaller than a certain defined threshold or a maximum number of iterations is reached, then the algorithm stops. Otherwise, go to step 3).

Note that in our iterative algorithm the first iteration, i.e., $k = 1$, can be considered as an initialization, since only for $k = 2$ some information about the noise is taken into account.

Similarly to the I-S-GSVD, which is the application of the S-GSVD in a iterative way, the same procedure can be applied for the tensor prewhitening scheme called n-mode products with matrix inversions proposed in [1]. Note that although we call the proposed scheme I-S-GSVD, we consider the S-GSVD II also proposed in [1] in the simulations instead of the S-GSVD. The S-GSVD II is computationally more expensive, however, it has a better a accuracy in comparison with the S-GSVD.

In our simulations we have observed that between two and three iterations are always sufficient to achieve the convergence.

IV. Simulation results

Here we generate our samples based on the data model of (1), where the spatial frequencies $\mu_i^{(r)}$ are drawn from a uniform distribution in $[-\pi, \pi]$. The source symbols are zero mean i.i.d. circularly symmetric complex Gaussian distributed with power equal to σ_s^2 for all the sources. The SNR at the receiver is defined as $\mathrm{SNR} = 10 \cdot \log_{10}\left(\frac{\sigma_s^2}{\sigma_n^2}\right)$, where σ_n^2 is the variance of the elements of the white noise tensor $\boldsymbol{\mathcal{N}}$ in (2).

Here we consider that the elements of the noise covariance matrix in the i-th mode $\boldsymbol{W}_i = \boldsymbol{L}_i \cdot \boldsymbol{L}_i^{\mathrm{H}}$ vary as a function of the correlation coefficient ρ_i similarly as in [1]. As an example we consider in (5) the structure of $\boldsymbol{W}_i$ as a function of ρ_i for $M_i = 3$

$$\boldsymbol{W}_i = \begin{bmatrix} 1 & \rho_i^* & (\rho_i^*)^2 \\ \rho_i & 1 & \rho_i^* \\ \rho_i^2 & \rho_i & 1 \end{bmatrix}, \qquad (5)$$

where ρ_i is the correlation coefficient. Note that also other types of correlation models can be used. To be consistent with (4), we normalize $\boldsymbol{L}_i$ such that $\mathrm{tr}(\boldsymbol{L}_i \cdot \boldsymbol{L}_i^{\mathrm{H}}) = M_i$.

In order to compare the performance of the proposed prewhitening scheme combined with R-D Standard-Tensor ESPRIT (R-D STE), we compute the total RMSE of the estimated spatial frequencies $\hat{\mu}_i^{(r)}$ as follows

$$\mathrm{RMSE} = \sqrt{\mathrm{E}\left\{\sum_{r=1}^{R}\sum_{i=1}^{d}\left(\hat{\mu}_i^{(r)} - \mu_i^{(r)}\right)^2\right\}}. \qquad (6)$$

In all figures, the same legend is used, where R-D STE w/o PWT means R-D STE without any prewhitening. R-D STE I-S-GSVD stands for the R-D STE combined with S-GSVD II [1], while the unknown noise is estimated deterministically with the iterative approach proposed in Section III. R-D STE S-GSVD stands for the R-D STE applied together with the S-GSVD II [1] and assuming that N samples in the absence of signal components are available.

From Fig. 1 to Fig. 7, we consider an array size of $M_r = 5$ for $r = 1, \ldots, 5$, and $N = 5$ snapshots, while for Figs. 8 and 9, we consider an array size $M_r = 5$ for $r = 1, \ldots, 7$, and $N = 7$ snapshots.

By comparing the RMSE using the I-S-GSVD without any information of the noise statistics to the RMSE using the S-GSVD which requires second order statistics of the noise in Fig. 1, the difference is insignificant. In Fig. 2, we consider the same scenario as in Fig. 1, but we evaluate the SNR versus the number of iterations k and we fix the noise correlation coefficients ρ_i to 0.9. For $K = 2$ iterations, a performance very close to the converged RMSE is obtained. Therefore, in Fig 1, we have considered $K = 2$.

From Fig. 1 to Fig. 3, we reduce the SNR from 20 dB to 5 dB and the number of maximum iterations is increased from $K = 2$ to $K = 3$ such that the RMSE of the I-S-GSVD is closer the converged RMSE. Moreover, while in Fig. 2 the curves of the I-S-GSVD and the S-GSVD are very close to each other, in Fig. 4 there is a very small gap. Therefore, by reducing the SNR, the performance of the I-S-GSVD becomes closer to the performance of using no prewhitening scheme. Therefore, we can state that for high and intermediate SNR regimes, the I-S-GSVD and the S-GSVD have a similar performance. In addition, the proposed I-S-GSVD outperforms significantly the estimation without prewhitening for intermediate and high noise correlation levels as exemplified in Fig. 5, where the total RMSE is computed by varying the SNR from -35 dB to 40 dB.

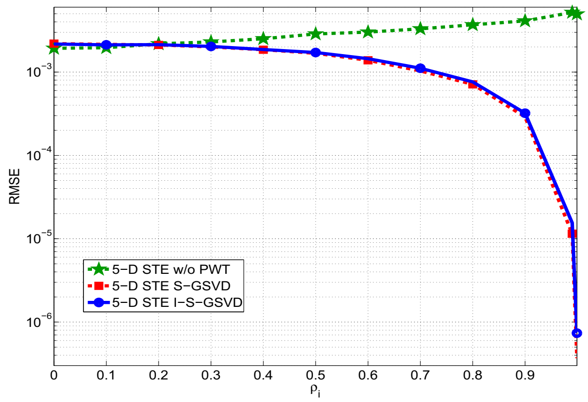

Fig. 1. Total RMSE of the 5 estimated spatial frequencies versus ρ_i for $i = 1, \ldots, 5$ is depicted. The SNR and the number of sources d are set to 20 dB and 3, respectively. The array size is $M_i = 5$, for $i = 1, \ldots, 5$, and $N = 5$. The I-S-GSVD is stopped after the second iteration, i.e., $K = 2$.

Also comparing Figs. 1 and 3, where the SNR is changed from 20 dB to 5 dB, the improvement obtained by the S-I-GSVD compared to the scheme without prewhitening increases. For instance, at $\rho_i = 0.9$, the RMSE of the S-I-GSVD is 10 times smaller than the STE without prewhitening in Fig. 1, while in Fig. 3 it is 20 times smaller.

In Figs. 6 and 7, we reduce the number of sources from 3 to 2 compared to Fig. 4. In this case, the gain of the I-S-GSVD with respect to the case without prewhitening is more significant. Such a behavior is also seen in [1] for the S-GSVD.

In Figs. 8 and 9, we increase the array size M_i, where $i = 1, \ldots, 5$, and the number of snapshots compared to the previous figures from 5 to 7. In particular, comparing Figs. 2 and 9, where the only difference is the increase of the tensor size, the gap between the I-S-GSVD and the S-GSVD remains.

V. Conclusions

In this paper, we propose the Iterative Sequential GSVD (I-S-GSVD) based prewhitening, which takes into account the Kronecker tensor structure of the colored noise and is applicable even when measurements without the desired signal components are not available. Our multidimensional prewhitening scheme has a similar performance as the S-GSVD, where measurements without the desired signal components are required. Moreover, besides the low computational complexity inherited from the S-GSVD, the I-S-GSVD achieves convergence with only two or three iterations, depending on the scenario.

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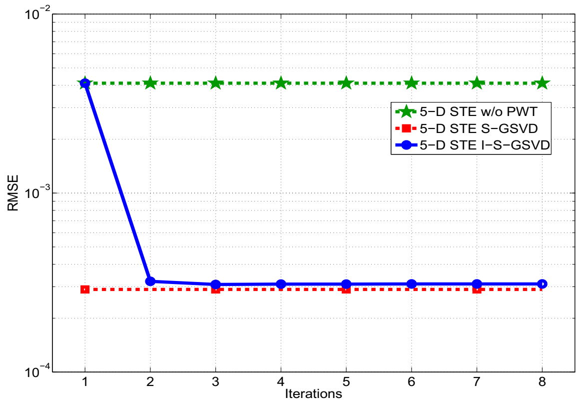

Fig. 2. Total RMSE of the 5 estimated spatial frequencies versus the number of iterations k is depicted. The SNR and the number of sources d are set to 20 dB and 3, respectively. The array size is $M_i = 5$, for $i = 1, \ldots, 5$, and $N = 5$. The noise correlation ρ_i is equal to 0.9. The same scenarios as in Fig. 1 is considered.

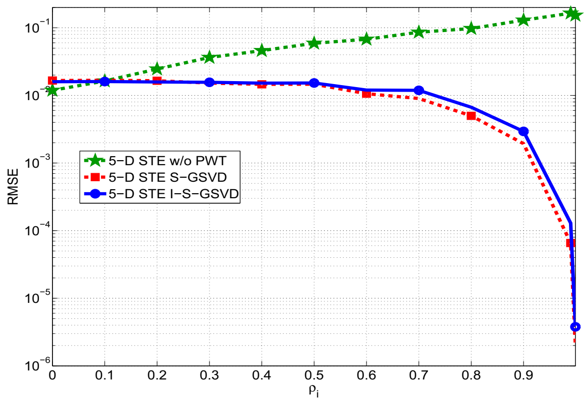

Fig. 3. Total RMSE of the 5 estimated spatial frequencies versus ρ_i for $i = 1, \ldots, 5$ is depicted. The SNR and the number of sources d are set to 5 dB and 3, respectively. The array size is $M_i = 5$, for $i = 1, \ldots, 5$, and $N = 5$. The I-S-GSVD is stopped after the third iteration, i.e., $K = 3$.

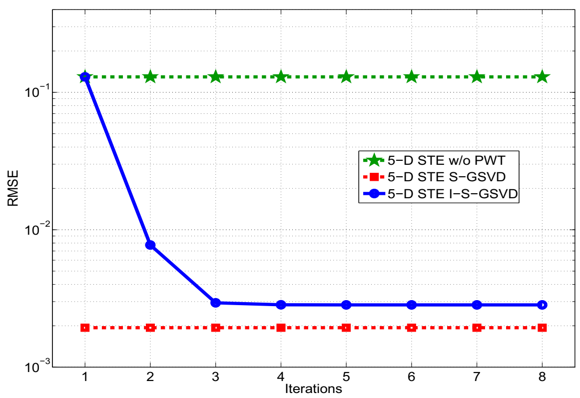

Fig. 4. Total RMSE of the 5 estimated spatial frequencies versus the number of iterations k is depicted. The SNR and the number of sources d are set to 5 dB and 3, respectively. The array size is $M_i = 5$, for $i = 1, \ldots, 5$, and $N = 5$. The noise correlation ρ_i is equal to 0.9. The same scenario as in Fig. 3 is considered.

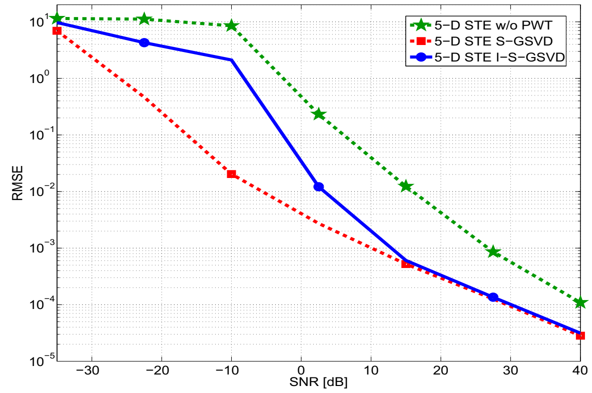

Fig. 5. Total RMSE of the 5 estimated spatial frequencies versus the SNR is depicted. The number of sources d and the correlation coefficient ρ_i are set to 3 and to 0.9, respectively. The array size is $M_i = 5$, for $i = 1, \ldots, 5$, and $N = 5$. The I-S-GSVD is stopped after the third iteration, i.e., $K = 3$.

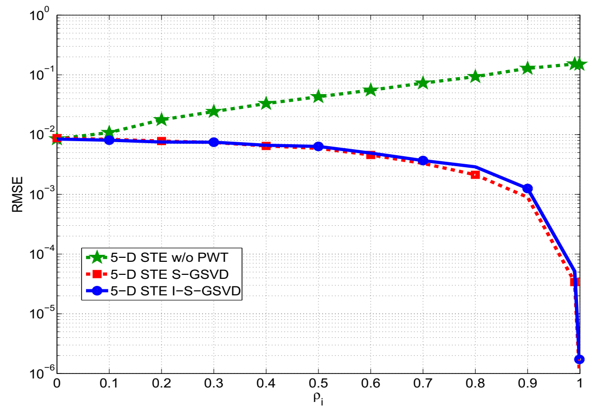

Fig. 6. Total RMSE of the 5 estimated spatial frequencies versus ρ_i for $i = 1, \ldots, 5$ is depicted. The SNR and the number of sources d are set to 5 dB and 2, respectively. The array size is $M_i = 5$, for $i = 1, \ldots, 5$, and $N = 5$. The I-S-GSVD is stopped after the third iteration, i.e., $K = 3$.

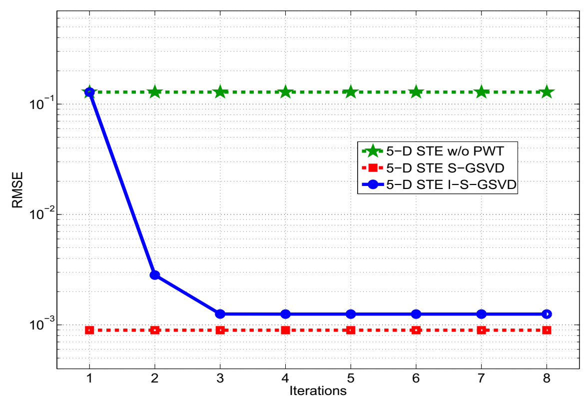

Fig. 7. Total RMSE of the 5 estimated spatial frequencies versus the number of iterations k is depicted. The SNR and the number of sources d are set to 5 dB and 2, respectively. The array size is $M_i = 5$, for $i = 1, \ldots, 5$, and $N = 5$. The noise correlation ρ_i is equal to 0.9. The same scenario as in Fig. 6 is considered.

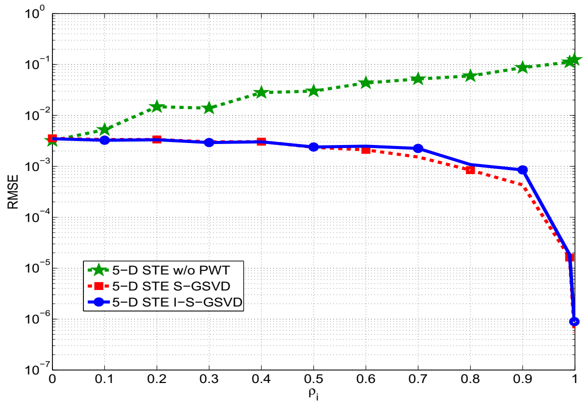

Fig. 8. Total RMSE of the 5 estimated spatial frequencies versus ρ_i for $i = 1, \ldots, 5$ is depicted. The SNR and the number of sources d are set to 5 dB and 3, respectively. The array size is $M_i = 7$, for $i = 1, \ldots, 5$, and $N = 7$. The I-S-GSVD is stopped after the third iteration, i.e., $K = 3$.

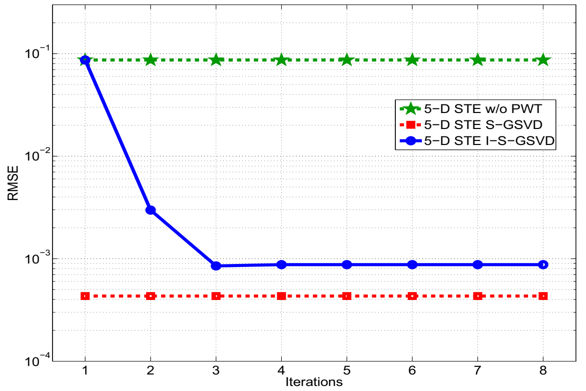

Fig. 9. Total RMSE of the 5 estimated spatial frequencies versus the number of iterations k is depicted. The SNR and the number of sources d are set to 5 dB and 3, respectively. The array size is $M_i = 7$, for $i = 1, \ldots, 5$, and $N = 7$. The noise correlation ρ_i is equal to 0.9. The same scenario as in Fig. 8 is considered.

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