Geometric Aspects of Polar and Near Polar Circular Orbits for the Use of Intersatellite Links for Global Communication

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ABSTRACT

In this paper the polar and the near polar circular orbit constellation are examined for the use of intersatellite links. We do the following definitions: The polar orbit has an inclination angle of exact 90 deg and the near polar orbit constellation has an inclination angle between 80 deg and 100 deg but unequal 90 deg. We present the exact equations for the intersatellite distance, the azimuth and elevation angles between two satellites. These equations are applied on a example system, a system that is using intersatellite links and that is based on the circular polar and near polar orbit constellation. To establish an intersatellite link, the distance between two satellites should be small and the variation of the azimuth and elevation angle should also be small. It is shown how a change of the inclination angle influences the whole orbit constellation in order to receive full coverage of the earth and it is shown how the inclination angle between two satellites on adjacent orbital planes influences the distance, the azimuth, and the elevation angle difference.

I. INTRODUCTION

In the last decades a great number of satellite systems for geostationary earth orbit (GEO) have been used. In the next few years mobile satellite systems for low earth orbit (LEO) and for medium earth orbit (MEO) will be available. These systems will provide worldwide telecommunications. The names of the most important first proposed systems are Iridium [4], Globalstar [12], ICO [5], and Teledesic [11]. Since the year 1996/1997 some additional systems for high speed data communication over satellite have been proposed, the important systems are M-Star [7], Celestri [8], and SkyBridge [10]. Some of these systems (Iridium, Teledesic, M-Star, Celestri) will use intersatellite links (=direct connections between the satellites), others will not. For systems with ISLs the messages are routed through direct connections between satellites in the same and in adjacent orbital planes. Using fast intersatellite connections, the long echo associated with geostationary links will be avoided. The connections can be established with Inter-Orbit ISLs and with Intra-Orbit ISLs:

(1) Intra-Orbit ISLs connect satellites in the same orbital plane, each satellite in an orbital plane establishes a link to a satellite flying ahead or behind itsself. Since the relative motion of the satellites in the same orbital plane is close to zero, the antenna pointing angles of these ISLs are constant and no antenna steering is necessary.

(2) Inter-Orbit ISLs. Inter-Orbit ISLs are connections between satellites in adjacent orbital planes. Because of the relative motion between the satellites, two satellites in adjacent orbital planes see each other under time-varying distance, azimuth and elevation angles. Hence, antenna steering is necessary.

In order to establish intersatellite links, it is necessary to calculate the exact positions of the satellites and to optimize the system in such a way, that the distance, the azimuth and the elevation angle between two satellites, that are connected via ISLs, do not vary too much.

In this paper we consider satellites in a circular polar orbit constellation [9] and near polar constellation. All satellites have the same altitude and each orbital plane has the same inclination angle.

In the first section of this paper we describe the circular polar and near polar orbit constellation. We show how the satellites are distributed around the earth and how the constellation is calculated to archive global coverage.

In the second part of the paper we show the effects of changing the inclination angle on the distance, and on the azimuth and elevation angles of the ISLs. We derive common equations for the distance and for the azimuth and elevation angles between two satellites as a function of the orbit parameters of the circular polar and near polar orbit constellation.

II. THE CIRCULAR POLAR AND THE NEAR POLAR ORBIT CONSTELLATION

In a polar orbit constellation and in a near polar orbit constellation each orbital plane has the same number of satellites, the same inclination, and all satellites have the same altitude. The value of the inclination angle i is 90 degrees (polar orbits) or near 90 degrees (near polar orbits, between 80 ...100 degrees). The satellites of the circular polar orbit constellations are moving in northerly direction on one side of the earth and in southerly direction on the other side of the earth. Due to this constellation there are two regions where satellites in adjacent orbital planes move in different directions, in [9] called counter-rotating interface. When the satellites in two adjacent orbital planes are moving in the same relative direction along the coverage interface, the interface is called co-rotating interface [9]. In order to ensure continious coverage along the interfaces, the angle separation Δ_2 between the satellites on the counter-rotating interface must be different than the angle separation Δ_1 for satellites on the co-rotating interface. In Fig. 1 polar view of the co- and counter-rotating interfaces with the different angle separations are shown.

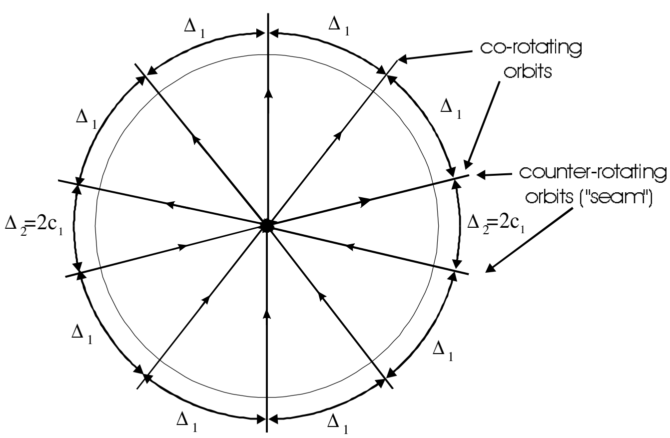
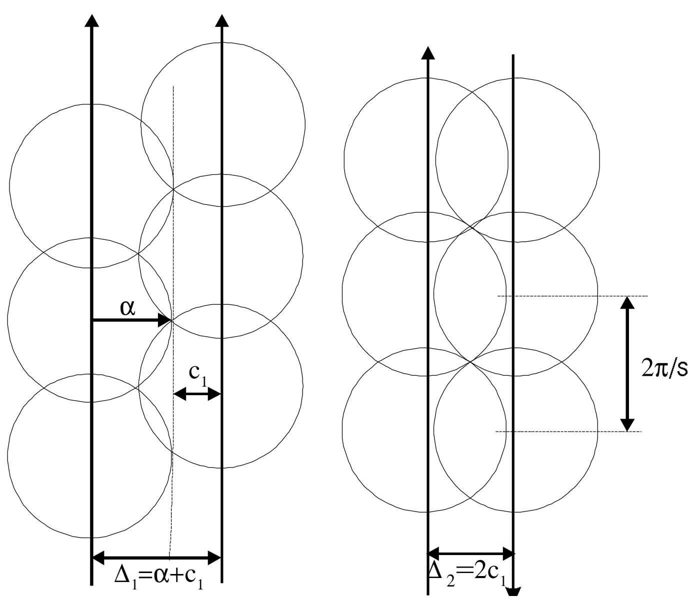


Fig 1 Polar view on the co- and counter-rotating orbits and the seam



a) co-rotating b) counter-rotating

Fig 2 Geometry of the coverage of adjacent orbital planes

The angular separations for the polar orbit constellation (i = 90°) (Fig. 1 and 2) are

$$\Delta_1 = c_1 + \varepsilon \tag{1}$$

$$\Delta_2 = 2c_1 \tag{2}$$

where c_1 is (for single coverage)

$$c = \arccos\left(\cos\alpha / \cos(\pi/s)\right) \tag{3}$$

and s is the number of satellites in each orbital plane. The earth central angle α is related to the elevation constraint E by

$$\alpha = \arccos\left[(R_e/(R_e+h))\cos E\right] - E \tag{4}$$

where R_e is the earth radius and h is the altitude of the satellites over the earth surface. For the circular polar satellite constellation we get 2(p-1) co-rotating interfaces and two counter-rotating interfaces which results in the equation

$$2(p-1)\Delta_1 + 2\Delta_2 = 2\pi \tag{5}$$

and with equations 1-3 in

$$(p+1)\arccos\left(\frac{\cos\alpha}{\cos\frac{\pi}{s}}\right) + (p-1)\arccos\left(\frac{\cos\alpha}{\cos\left(\frac{\pi}{s}-\beta\right)}\right) = \pi \tag{6}$$

Given a minimum elevation angle E, these equations can be solved for α and then for c_1, Δ_1, and Δ_2 with given values for p, and s. In Tab 1 the solutions for α and Δ_1 of different constellations for given p (p = 2...6) and s for the circular polar orbit constellation (i = 90°) are listed.

p	s	α [°]	Δ_1 [°]	h [km], E=10°
2	3	66.716312	104.477689	20958.620810
2	4	57.631603	98.421103	10127.101222
2	5	53.219432	95.479652	7562.365855
3	5	42.279402	66.139862	3888.488654
3	6	38.682256	64.341277	3135.514671
3	7	36.450544	63.225432	2738.578966
4	7	30.781856	48.312586	1917.191788
4	8	28.913101	47.565071	1694.402093
4	9	27.607368	47.043199	1550.606106
5	9	24.181000	38.060272	1214.622688
5	10	23.045540	37.681845	1115.251907
5	11	22.194174	37.398170	1044.325151
6	11	19.906922	31.401933	868.026177
6	12	19.146507	31.184615	813.760184

Table 1 Angle values of different polar orbit constellations (i = 90°) with single coverage

p	s	α [°]	Δ_1 [°]	*h* [km], *E*=10°
2	3	66.268636	106.660652	22123.695303
2	4	58.803820	101.705729	10994.351932
2	5	55.122973	100.147445	8553.300302
3	5	42.279402	66.139862	3888.488620
3	6	38.682257	64.341277	3135.514644
3	7	36.450544	63.225432	2738.578966
4	7	31.012491	48.974857	1946.163988
4	8	29.139704	48.160696	1729.321720
4	9	27.831431	47.597872	1574.623733
5	9	24.521583	39.010910	1245.552758
5	10	23.380174	38.555620	1143.959763
5	11	22.524771	38.220791	1071.511689
6	11	20.339753	32.589297	899.857472
6	12	19.568797	32.283361	843.638459

Table 2a Angle values of different near polar orbit constellations (i = 80°) with single coverage

p	s	α [°]	Δ_1 [°]	*h* [km], *E*=10°
2	3	66.856398	105.036278	21244.765243
2	4	57.933649	99.273283	10341.512810
2	5	53.719206	96.712411	7808.076687
3	5	42.279402	66.139862	3888.488620
3	6	38.682257	64.341277	3135.514644
3	7	36.450544	63.225432	2738.578966
4	7	30.835106	48.466441	1923.851101
4	8	28.965793	47.704283	1700.403028
4	9	27.659622	47.172815	1556.183452
5	9	24.256283	38.272085	1221.410381
5	10	23.120364	37.878251	1121.629681
5	11	22.268681	37.584303	1050.413285
6	11	19.996083	31.649290	874.526675
6	12	19.235053	31.417147	819.972171

Table 2b Angle values of different near polar orbit constellations (i = 85°) with single coverage

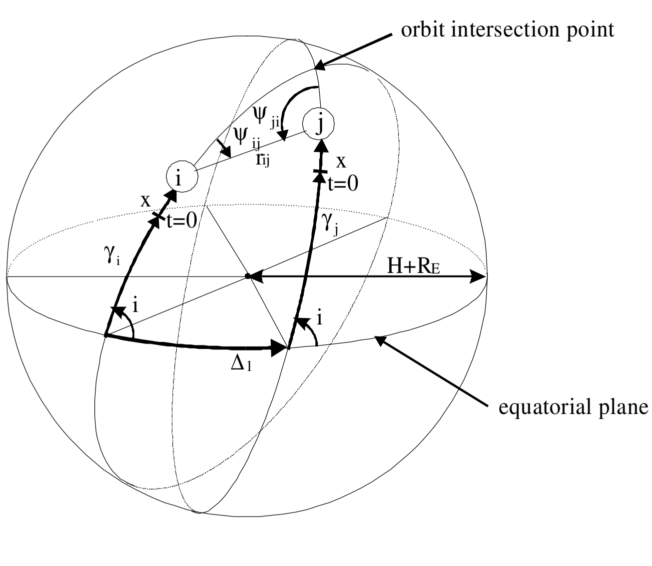


Fig 3 Geometry of a satellite pair [3].

The near polar orbit constellation can be calculated from the polar orbit constellation by the use of an inclination angle different to 90°. The inclination angle of the near polar orbit constellation is between 80 deg and 100 deg. The near polar orbit constellation also enables global coverage of the whole earth. The equations of the polar orbit constellation must be expanded with the inclination angle. Over the equator we have, again, 2(p-1) co-rotating interfaces and two counter-rotating interfaces which results in the equation for global coverage of the whole earth

$$2(p-1)\Delta_1(i) + 2\Delta_2(i) = 2\pi \tag{7}$$

with the angular separations $\Delta_1(i)$ and $\Delta_2(i)$ of the near polar orbit constellation, which are dependent on the inclination i

$$\Delta_1(i) = \arcsin\left(\frac{\sin\Delta_{1i}}{\sin i}\right) \tag{8}$$

$$\Delta_2(i) = \arccos\left(\frac{\cos\Delta_{2i} + \cos^2 i}{\sin^2 i}\right) \tag{9}$$

Finally, with equations 7-9 and 1-3 we get

$$(p-1)\arcsin\left(\frac{\sin\left[\alpha + \arccos\left(\frac{\cos\alpha}{\cos\pi/s}\right)\right]}{\sin i}\right) +$$
$$\arccos\left(\frac{\cos\left[2\arccos\left(\frac{\cos\alpha}{\cos\pi/s}\right)\right] + \cos^2 i}{\sin^2 i}\right) + \tag{10}$$

This equation can be solved for α and then for c_1, Δ_{1i}, and Δ_{2i} with given values for p, s, and i. In Tab 2 the solutions for α and Δ_1 of different constellations for given p (p = 2...6) and s for two near polar orbit constellations (a) i = 80°, b) i = 85°) are listed. All the constellations in Tab 1 and 2 provide global coverage with a minimal elevation angle of E = 10 deg.

We also have to change the phasing angle difference between two satellites on adjacent orbital planes. For the polar orbit constellation we have the phasing angle difference

$$\Delta\gamma = \gamma_j - \gamma_i = 180°/s \tag{11a}$$

For the near polar orbit constellation we must consider the inclination angle, which results in the equation

$$\Delta\gamma(i) = \gamma_j(i) - \gamma_i(i) = 180°/s - \arctan(\cos i \tan\Delta\gamma_1(i)) \tag{11b}$$

It can be seen (Tab 1+2) that the angular separation Δ_1 and the altitude h is increasing with smaller inclination angles.

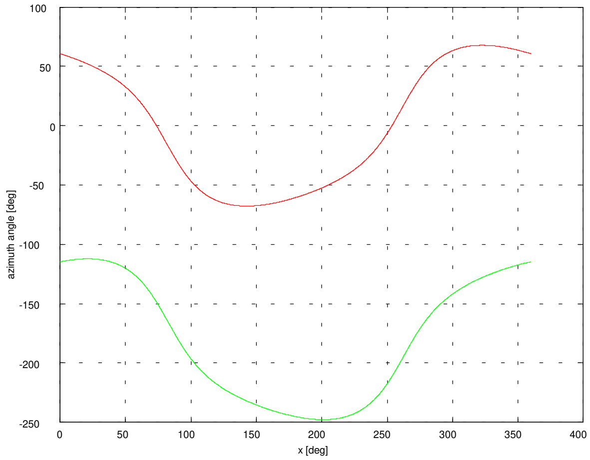


Fig 4 Azimuth angles between satellites i and satellites j and vice versa as a function of time for $i = 90°$

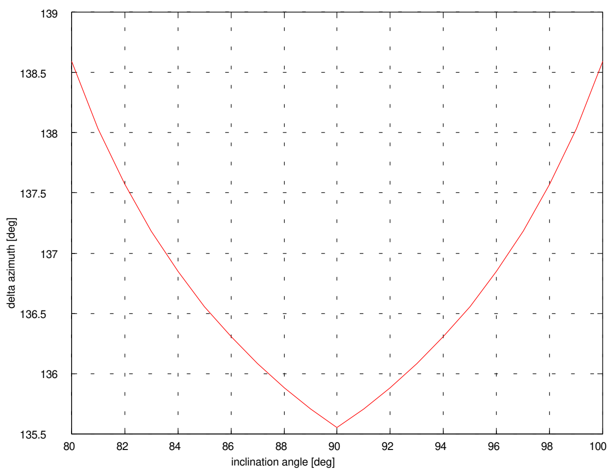


Fig 5 Azimuth angle difference between satellites i and satellites j as a function of the inclination angle for the named constellation

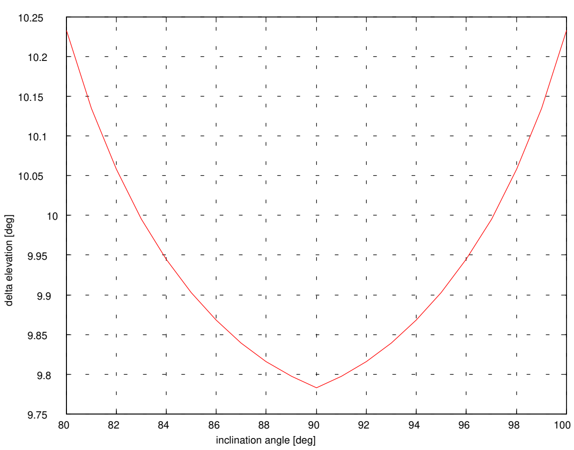


Fig 6 Elevation angle difference between satellite i and satellite j as a function of the inclination angle for the named constellation

II. INTERSATELLITE LINKS

Intersatellite links are direct connections between satellites. In order to examine the intersatellite links between two satellites we must derive the equations of the distance, the azimuth angle, and the elevation angle of two satellites as a function of orbital parameters. In Fig. 3 the geometry of a satellite pair is shown. The phase angle of satellite i is γ_i and of satellite j it is γ_j. The azimuth angle from satellite i to satellite j is ψ_{ij} and from satellite j to satellite i it is ψ_{ji}. The inclination angle of the two orbital planes is i and the intersatellite range is r_{ij}. The intersatellite great circle range r_{ij} can be derived applying the law of cosines for sides to the small triangle formed by the two satellites i and j and the orbit intersection point [3].

$$\begin{aligned}\cos r_{ij} = (&\cos^2(\Delta_1/2) - \cos^2 i \sin^2(\Delta_1/2)\cos(\gamma_i - \gamma_j) + \\ &+ 2\cos i \sin(\Delta_1/2)\cos(\Delta_1/2)\sin(\gamma_i - \gamma_j) - \\ &- \sin^2 i \sin^2(\Delta_1/2)\cos(\gamma_i + \gamma_j + 2x)\end{aligned} \tag{12}$$

The elevation angle between the two satellites can be derived using the intersatellite great circle range. It results in

$$\theta_{ij} = r_{ij} / 2 \tag{13}$$

The distance is then

$$d_{ij} = 2\,(h + R_e)\sin(r_{ij} / 2) \tag{14}$$

The equations for the azimuth angle can be calculated using the Napier analogies

$$\begin{aligned}\psi_{ij} = &[\sin i \sin(\Delta_1/2)\cos(\gamma_j + x) - \sin 2i \sin^2(\Delta_1/2)] / \\ &/ [\sin^2 i \sin^2(\Delta_1/2)\sin(\gamma_i + \gamma_j + 2x) + \\ &+ 2\cos i \cos(\Delta_1/2)\sin(\Delta_1/2)\cos(\gamma_j - \gamma_i) + \\ &+ (\cos^2(\Delta_1/2) + \cos^2 i \sin^2(\Delta_1/2))\sin(\gamma_j - \gamma_i)]\end{aligned} \tag{15}$$

The equation of the azimuth angle for looking from satellite j to satellite i can be obtained by an interchange of indices.
For the polar orbit constellation with an inclination angle of i = 90 deg the equations can be simplified and we get

$$\cos r_{ij} = (\cos^2(\Delta_1/2)\cos(\gamma_i - \gamma_j) + \sin^2(\Delta_1/2)\cos(\gamma_i + \gamma_j + 2x) \tag{16}$$

and

$$\begin{aligned}\psi_{ij} = &[\sin(\Delta_1/2)\cos(\gamma_j + x)] / \\ &/ [\sin^2(\Delta_1/2)\sin(\gamma_i + \gamma_j + 2x) + \cos^2(\Delta_1/2)]\end{aligned} \tag{17}$$

Now we consider the constellation with 66 satellites in 6 orbital planes for inclination angles between 80 deg and 100 deg.

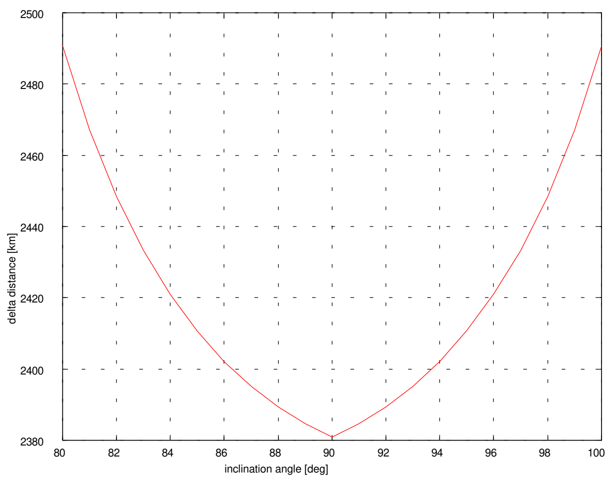


Fig 7 Distance difference between satellite i and satellite j as a function of the inclination angle for the named constellation for the named constellation

To enable a connection between satellites the variation of the distance, the azimuth angle, and the elevation angles should be not too large in order to decrease the antenna steering.

i) Azimuth angle

First we calculate the azimuth angles between two satellites on the co-rotating interface for this constellation. The azimuth angles between satellite i looking to satellite j for this constellation is illustrated in Fig. 4 as a function of time. Due to the time-variant behaviour of a constellation the variations of the azimuth angles are large. In Fig 5 the maximal change of the azimuth angle is shown. It can be seen that the maximal change of the azimuth angle is increasing with inclination angles greater or smaller than 90 deg. In the next step we investigate the elevation angle and the distance for these different inclination angles.

ii) Elevation angle

In Fig. 6 the difference (delta) change of the elevation angle is shown as a function of the inclination angle. In the picture it can be seen, that the variation of the elevation angle is also increasing with an inclination angle smaller or greater 90 deg (Fig. 6).

iii) Distance

The difference distance of satellite i looking to satellite j is shown in Fig. 7. The difference distance has the same behaviour as the difference elevation. It can be seen, that the variation of the distance is increasing with changing the inclination angle different to 90 deg. (Fig. 7)

IV. CONCLUSION

This paper has shown that the variation of the elevation and azimuth angle, and the distance is increasing with an inclination angle different to 90 deg, this occurs due to the higher altitude and the changing phase angle difference This means that the constellations are getting worse for inclination angles unequal to 90 deg. In [6] the same constellation is examined for not changing the altitude and the phase angle difference. The result of [6] is that the azimuth angle, the elevation angle, and the distance is decreasing. But such a constellation is not optimal for coverage and the probability of visibility is decreasing under 100% for an inclination angle smaller 86 deg or greater 94 deg.

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