EINLADUNG

ZUM MATHEMATISCHEN KOLLOQUIUM

Es spricht

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Zum Thema:

Nonlinear balanced truncation via infinite-dimensional Koopman lifting

Abstract:

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Balanced truncation is a well-known, powerful method for system theoretic model reduction. Initially been proposed for linear control systems, nonlinear generalizations exist but require solutions to two nonlinear PDEs of Hamilton-Jacobi type. On the other hand, by employing a Koopman (or composition) lifting, the evolution of a nonlinear ODE can be connected to a linear system acting on an infinite-dimensional function space. In [2], a specific output energy functional for the nonlinear system

$$\dot{x}(t) = f\big(x(t)\big), \;\; x(0) = z \tag{1}$$

has been related to a quadratic energy functional for the infinite-dimensional linear system

$$\psi(t) = \mathcal{A}\psi(t), \;\; \psi(0) = \delta_z \tag{2}$$

where the distributional initial condition is understood formally as the limit of a sequence of appropriate mollifiers. If considered on a suitably chosen weighted Lebesgue space, the resulting Koopman system can be shown to be exponentially stable. In [2], this has been used to discuss the solution $\mathcal{Q}$ of the following operator Lyapunov equation

$$\langle \mathcal{A}\varphi, \psi \rangle_{\mathcal{Q}} + \langle \varphi, \mathcal{A}\psi \rangle_{\mathcal{Q}} + \langle \mathcal{C}\varphi, \mathcal{C}\psi \rangle_{\ell^2} = 0 \qquad \forall \varphi, \psi \in \mathcal{D}(A) \tag{3}$$

for a class of output operators $\mathcal{C}$. In this talk, equation (3) serves as the starting point for an infinite-dimensional balanced truncation strategy similar to [3, 4, 5]. Our interest however is to approximate the nonlinear mapping $z \mapsto x(t)$ characterized by (1). For this, a second operator Lyapunov equation is introduced and by means of an output-normalized realization of (2), a truncated reduced-order model is obtained. This model is shown to satisfy an $\mathcal{H}_2$-type error bound as in [1] which can even be obtained a priori. The result is a linear finite-dimensional system which approximates (1) for z drawn from a probability distribution.

References:

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[2.] T. Breiten and B. Höveler. On the approximability of Koopman-based operator Lyapunov equations. *SIAM J. Control Optim.*, 61(5):3131–3155, 2023.
[3.] K. Glover, R. Curtain, and J. Partington. Realisation and approximation of linear infinitedimensional systems with error bounds. *SIAM J. Control Optim.*, 26(4):863–898, 1988.
[4.] C. Guiver and M. Opmeer. Model reduction by balanced truncation for systems with nuclear Hankel operators. *SIAM J. Control Optim.*, 52(2):1366-1401, 2014.
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Donnerstag, 11. Juli 2024, 15:00 Uhr, C 113

(Kaffee & Kekse, 14:30 Uhr, C 325)

Alle Interessierten sind herzlich eingeladen!

Ilmenau, 12.06.2024 — Das Institut für Mathematik